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(M, S)-OPTIMALITY CRITERION FOR REGULAR FRACTIONAL FACTORIAL DESIGNS

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ABSTRACT

The performance of (M, S)-optimality criterion on Regular two-level fractional factorial designs (FFD's) is studied. The criterion has good estimation capacity and easy to compute. In the past, researchers have used (M, S)-optimality criterion on Regular two-level fractional factorial designs up to two factor interactions (2*fi*'s). This study is the extension of previous work and (M, S)-optimality criterion is used on Regular two-level FFD's to two factor interactions (2*fi*'s) as well as three factor interactions (3*fi's*). Models for three factor interactions are used to determine maximum trace C_d *and* minimum trace C_d^2 for the best designs. New formulae for two factor interactions and three factors interactions are developed and used. Two components of the (M, S) criterion, i.e., trace C_d and trace C_d^2 are derived as explicit functions of the numbers of three- and four-letter words. Generally, (M, S)-optimal designs are not Minimum Aberration MA designs but all MA designs up to 64 runs are (M, S)-optimal. Although main effects, 2*fi*'s and 3*fi*'s are focused in this study, but the (M, S) criterion is to be used when 3*fi*s or more than three factors are available.

Keywords: Alias sets, fractional factorial designs, minimum aberration, (M, S)-Optimality, regular fractional factorial design.

INTRODUCTION

According to FFDs it can be mainly divided into two portions regular designs and non-regular designs. Those designs which are made through different factors they are called regular designs. Combined belongings are also orthogonal or fully aliased while there run size is also a power of 2 or of *s*, generally, as those fractions of 2^n or s^n full factorial designs; while a multiple of 4 is the basic rule for the construction of non-regular designs, e.g. 4, 8, 12. In regular designs, the gaps between possible run size is more as the power increases. So the regular designs are not run size economical, particularly when the runs are costly to do.

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The most commonly used criterion which is called MA criterion is used for choosing a good regular design (Fries and Hunter, 1980).

When we assigned additional factors to Full Fractional Factorial Designs, they are called higher order interactions of a full factorial design. In this way, the researchers have an ability to add more factors by sacrificing to evaluate higher order relations. The guess of the major outcome is complete to be interchangeable from the estimate of the interaction column that were to which it was provided in which reason which was the two effects are fully aliased. In this way a particular run size and a fixed number of factors, Fractional Factorial Designs can be used if possible. However, the selection of set from good designs are most important. The concept of resolution criterion to rank the many possible FFD's is introduced by Box and Hunter, while Fries and Hunter (1980) reported that MA criterion is a refinement of Box and Hunter's criterion.

According to Shah (1960) who worked on this method that will hereafter be called *S*-optimality. In *S*-optimality, minimize $\sum_i \lambda_i^2$ where λ_i are nonzero eigenvalues (i =1, 2…) if the trace of information matrices of the competing designs are identical. The corresponding optimum design will be referred to as *S*-optimum. The (M, S)-optimality criterion was launched by Eccleston and Hedayat (1974) as a generalization of the *S*-optimality criterion of Shah (1960). It was a two-stage optimization process as follows. Let M denote the subclass of designs $N \in D$ such that the *C*-matrices have maximal trace, denoted by trace C_d among the designs in D. A design $N \in D$ is said to be (M, S)-optimal if $N \in M$ and if the square of its C_d-information matix has minimum trace among the designs in *M.* To use the (M, S)-optimality criterion, first form a subclass of designs whose information matrices have maximum trace, then select designs from that subclass in this way the square of the information matrix has minimum trace. The resulting design is called the (M, S)-optimum design (Eccleston and Hedayat, 1974).

The (M, S)-optimality criterion is commonly used and supported by many authors these criteria are identified as the alphabetical optimality criteria. A major advancement in this regard was the equivalence theorem for *D*-optimum and *G*-optimum designs proved in Kiefer and Wolfowitz (1960). Optimality theory was the main subject of several textbooks in the early 70's. The major contributions can be attributed to Fedorov (1972).

METHODOLOGY

According to Eccleston and Hedayat (1974) who worked on (M, S) procedure which is available in optimal design literature. Nowadays, in agricultural and industrial experiments, there are many situations where three-factor interactions 3*fi*'s are also important. So, this study is an extension of Qu *et al.* (2008) focused on three-factor interactions. For this purpose we extended formula of two factor interaction of two-level design to three factor interactions of design *d* with *m* runs of *n* factors for $\binom{n}{2}$ two factor interactions and $\binom{n}{3}$ three-factor interactions, consider the following linear model.

$$E\,[z(y_1,..y_2,\ldots,y_n)] = \gamma_0 + \sum_{i=1}^{n} \gamma_i y_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \gamma_{ij} y_i y_j + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n} \gamma_{ijk}\, y_i y_j y_k \qquad (1)$$

where Z (y_1, y_2, … y_n) is the observed response of treatment (y_1, y_2, . . . , y_n), y_i is the different levels of factor i and takes the value −1 or 1 (i = 1,..n). γ_0 is the grand mean, γ_i is the major outcome of factor i, γ_{ij} is the two-factor interactions between factors i and j, and γ_{ijk} is the 3*fis* interaction among the factors i , j and k. All the four-factor or higher-order interactions are omitted in model (1). Model (1) can also be written in matrix form, i.e., $Z = Y_1\gamma_1 + Y_2\gamma_2 + \varepsilon$ (2)

where Z is an $N \times 1$ vector of observations, $Y_1 = (1_N, y_1, \cdots, y_n)$, 1_N denotes an $N \times 1$ vector of 1's, $\gamma_1^{'} = (\gamma_0, \gamma_1, .., \gamma_n)$ stand for the grand mean and the n main outcomes. γ_2 is the vector of $\binom{n}{2}$ 2*fi*s two-factor interaction parameters and the vector of $\binom{n}{3}$ three-factor interactions parameters, Y_2 is the equivalent coefficient matrix of 2*fi*'s and 3*fi*'s both, ε is a vector of free chance errors with mean 0 and regular variance σ^2. As a common, γ_1 can be any subset that is of main interest with γ_2 in place of minor parameters. in the statement of regularity of errors, the Fisher in rank matrix of γ_2 in step for γ_1 is $C_d = Y'_2 Y_2 - (Y_1^{'} Y_2)' (Y'_1 Y_1)^{-1}(Y'_1 Y_2)$. Since C_d is symmetric, it denotes $C_d^{'} C_d$ as trace C_d^2. The (M, S) criterion defines those designs which capitalize on trace C_d and then discovers designs within this subclass that reduce trace C_d^2 is called an (M, S)-optimal designs reported in Shah and Sinha (1989). For a 2^{n-k} regular plan with one duplicate, $Y'_1 Y_1 = 2^{n-k} I_{n+1}$ where I_{n+1} is the (n + 1)- multidimensional identity matrix. Thus, $C_d = Y'_2 Y_2 - 2^{-(n-k)} (Y'_1 Y_2)' (Y'_1 Y_2)$.

It is noted that although the (M, S) optimality criterion explained for the basis of model (2) and for 2 level FFDs, yet trace C_d and trace C_d^2 are, in fact, free of the option of orthogonal dissimilarities (Dey and Mukerjee 1999). Mixed-level and multi-level designs accept the application of (*M, S*) optimnality criterion. In the literature, it is shown that many statisticians also studied (*M, S*)-optimality in the FFD's. It is regarded as the joint order on γ_1 and γ_2 while this study is focused on the conditional information on γ_2 given γ_1. This criterion is very useful in this study, particularly in the situation in which main effects are of major portion, but the researcher would like to have a lot of details on 2*fi*'s and 3*fi*'s as likely below the proposition so as to 4*fis* and higher-order interactions are unimportant. The trace C_d and trace C_d^2 include not only the full details of 2*fi*'s and 3*fi's* on major outcomes but in order on 2*fi's* and 3*fi's* like wise.

Jacroux (2004) has elaborated the basis of resolution *III* or higher for regular designs. His mathematical framework withstands the designs of resolution *III* and *IV,* which provided basis that major outcomes can be aliased with 2*fi*'s. In case of regular FFDs of a 2^{n-k}, while design *d* would be W_i (*d*), the number of words of length *i* in the defining relation. Then *W* (*d*) = W1 (*d*),.W_n(*d*) belongs to design *d* of word length pattern. In case of comparison of two designs, say d_1 and d_2, the design is called MA designs which have less aberration. Mathematically the following conditions are assumed to be true: W_1 (d_1) ≠ W_1 (d_2) then W_1 (d_1) < W_1 (d_2).

In a 2^{n-k} of *d* of resolution *III* designs or more, (2^k−1) of the 2^n −1 factorial belongings show in the essential order. The left over (2^n–2^k) effects are divided into $g = 2^{n-k} - 1$ alias sets each of size 2^k, where *n* of the *g* alias sets include major outcomes (one each). Let *f* = *g-n* and the *f* alias sets not holding major outcomes be $M_1, \cdot\cdot, M_f$. Also let the *n* alias sets containing major outcomes be M_{f+1}, Mg. For $1 \le i \le g$, let $m_i(d)$ be the number of 2*fi*'s in M_i. Then

$$\sum_{i=f+1}^{g} m_i(d) = 3W_3(d) \qquad (3)$$

Equation (3) defines three 2*fi*'s which consist of $3W_3$ (*d*) in order that main effects are aliased with 3 two-factor interactions. Main effects are also aliased with three (2*fi*'s) with the word length of 3. This involves that $\binom{n}{2}-3W_3$(d) two-factor interactions that are not aliased with main effects. In the same way, let $m_j(d)$ be the number of 3*fi*'s in M_j, then

$$\sum_{i=f+1}^{g} m_j(d) = 4W_4(d) \qquad (4)$$

It is also clear from equation (4) that there are $4W_4$(*d*) four 3*fi*'s which are aliased with main effects because every factor of distance 4 word the important comparative identifies four three-factor interactions 3*fi*'s which are aliased with the main effects. In the same way, there are $\binom{n}{3}$ – $4W_4$(*d*) three-factor interactions 3*fi*'s which are not aliased with main effects, then the crosswise part of $(Y_2^{'}Y_2)'(Y_2^{'}Y_2)$ is 4^{n-k}. The designs elaborate that the defining relation has been developed in such way that the main effects are confounded with two-factor interactions 2*fi*'s and 0 otherwise. Two main effects cannot be confounded with the same 2*fi*'s and hence the design is said to be resolution *III* or higher. Entries of the upper and lower triangle of the matrix would be zero when main effects are confounded with two-factor interactions and the property of 4^{n-k} will not withstand when the same main effects are both confounded. Similarly there are $\binom{n}{3}$ - $4W_4$ (d) three-factor interactions which are not aliased with major belongings, then the diagonal element of $(Y_2' Y_2)'(Y_2' Y_2)$ is 4^{n-p} if the 2*fi*'s are aliased with a major effect, because the design resolution is at least *III*, so it means that they cannot be 2 major effects aliased with 2*fi's*. The off-diagonal element is zero if the 2*fi*'s are not aliased with a main effect and 4^{n-k} if they are both aliased with the same main effect. Likewise, the (*i, i*) th sloping part of $Y_2' Y_2$ is 2^{n-k}, and the (*i, j*)th off-diagonal part is zero if the *i* th and *j* th 2*fi*'s are not aliased with one another and 2^{n-k} if which is therefore, trace C_d is the same to the figure of 2*fi*'s that are not aliased with major belongings (i.e., 2*fi*'s in $M_1,.., M_f$) increased by 2^{n-p}, and

$$\text{Trace } C_d = 2^{n-k}\left[\sum_{i=1}^{f} mi(d)\right] = 2^{n-k}\left[\binom{n}{2} - 3w_3(d)\right]$$

$$\text{Trace } (C_d^2) = 4^{n-k}\left[\sum_{i=1}^{f} mi(d)^2\right] = 4^{n-k}\left[\binom{n}{2} - \sum_{i=f+1}^{g} mi(d)^2 + 6w_4(d)\right]$$

so, this purpose fractional factorial 2 level design *d* of resolution *III* or higher, our proposed formulae for trace C_d and trace C_d^2 for three-factor interactions are:

Trace

$$(C_d) = 2^{n-k}\left[\sum_{i=1}^{f} mi(d) + \sum_{j=1}^{k} mj(d)\right] = 2^{n-k}\left[\left\{\binom{n}{2} - 3w_3(d)\right\} + \left\{\binom{n}{3} - 4w_4(d) - w_3(d)\right\}\right].$$

$$\text{Trace } (C_d^2) = 4^{n-k}\left[\sum_{i=1}^{f}\sum_{j=1}^{g} 2mi(d)mj(d) + \sum_{i=1}^{f} mi(d)^2 + \sum_{j=1}^{g} mj(d)^2\right]$$

The above formulae shows that the (M, S) optimality method used on designs that maximizes $\sum_{i=1}^{f} mi(d)$ (two-factor interactions) (or, equivalently, minimizes $W_3(d)$ first and then minimizes $\sum_{i=1}^{f} mi(d)^2$, when the MA method minimizes W_3(d) in start and then minimizes $W_4(d)$ (or, homogeneously, minimizes $\sum_{i=1}^{g} mi(d)^2$). Table 1 shows the 11, 2_{III}^{8-3} designs with different generators. Among these 11 designs, design 1, 2, 4, 9 and 10 has same maximum C_d that is 2304. Among this set of designs, design 4 and 9 has minimum C_d^2 that is 235520. According to the criterion, design having minimum C_d^2 in the set of designs, having maximum C_d should be considered the best design. So, design 4 and 9 are the best designs in this case.

Table 1. 2_{III}^{8-3} designs in 32 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2_{III}^{8-3}	*F= ABE, G=ABDE, H=BCDE*	2304	241664
2_{III}^{8-3}	*F= BC, G=ABCD, H=CDE*	2304	241664
2_{III}^{8-3}	*F= ABC , G=CD, H=DE*	2176	217088
2_{III}^{8-3}	*F=ABCE, G=CD, H=CDE*	2304	235520
2_{III}^{8-3}	*F=BCE, G=AE, H=CDE*	2176	217088
2_{III}^{8-3}	*F=DE, G=CDE, H=BDE*	1920	196608
2_{III}^{8-3}	*F=AE, G=CDE, H=CD*	2048	227328
2_{III}^{8-3}	*F=AB, G=AE, H=AD*	1920	184320
2_{III}^{8-3}	*F=DE, G=ABCDE, H=AE*	2304	235520
2_{III}^{8-3}	*F=CD, G=BCE, H=AD*	2304	241664
2_{III}^{8-3}	*F=ACE, G=ABCE, H=CDE*	2176	217088

Table 2. 2_{iv}^{8-2} designs in 64 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2_{iv}^{8-2}	*G=BCD, H=ABEF*	5120	532480
2_{iv}^{8-2}	*G=DEF, H=ABEF*	5120	614400
2_{iv}^{8-2}	*G = ABC, H = CDE*	4864	630784
2_{iv}^{8-2}	*G = ABCDE, H = ACDEF*	5120	614400
2_{iv}^{8-2}	*G = ABCDEF, H =CDEF*	5120	614400
2_{iv}^{8-2}	*G = BDE, H = ADEF*	5120	614400
2_{iv}^{8-2}	*G = ABDE, H = CDE*	5120	614400
2_{iv}^{8-2}	*G = ADE, H = CDEF*	5120	614400
2_{iv}^{8-2}	*G = ADEF, H = CEF*	5120	614400
2_{iv}^{8-2}	*G = AEF, H = ADEF*	5120	614400
2_{iv}^{8-2}	*G = ACEF, H = CDF*	5120	614400

Table 2 also shows 11, 2_{iv}^{8-2} designs of resolution *IV* in 64 runs with different generators. Among these 11 designs, all designs have same maximum C_d that is 5120 except design 3 which has 4864. Among this set of designs, design 1 has minimum C_d^2 that is 532480. Since design 1 has maximum C_d as well as minimum C_d^2, so, design 1 is the best design among these designs.

Table 3. Best designs in8 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{4-1}	*D=ABC*	48	768
2^{5-2}	*D=ABC, E=BC*	64	2048
2^{6-3}	*D=ABC, E=,BC, F=AC*	80	4864
2^{7-4}	*D=ABC, E=BC, F=BC, G=AC*	80	10496

Table 4. Best designs in 16 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{5-1}	*E=ABCD*	320	10240
2^{6-2}	*E=ABCD, F=BD*	432	20736
2^{7-3}	*E=ABCD, F=BCD, G=ABC*	576	41984
2^{8-4}	*E=BCD, F=ACD, G=BCD, H=AC*	784	85248
2^{9-5}	*E=ABCD, F=ACD, G=BCD, H=AD, J=BD*	960	153600
2^{10-6}	*E=ABCD, F=AC, G=BCD, H=ACD, J=BC, K=ABD*	1040	216320
2^{11-7}	*E=ABD, F=ACD, G=ABCD, H=AB, J=BC, K=AD, L=BCD*	1088	295936
2^{12-8}	*E=BC, F=BD, G=AD, H=AD, J=AC, K=CD, L=AB, M=ABCD*	1136	494848
2^{13-9}	*E=ABC, F=ABD, G=CD, H=BD, J=CD, K=AB, L=AC, M=AD, N=BC*	1088	667648
2^{14-10}	*E=ABCD, F=ABC, G=ABD, H=ACD, J=BCD, K=AB, L=AC, M=AD, N=BC, O=BD*	560	313600
2^{15-11}	*E=ABCD, F=ABC, G=ABD, H=ACD, J=BCD, K=AB, L=AC, M=AD, N=BC, O=BD, P=CD*	0	0

Table 5. Best designs in 32 runs on the basis of maximum C_d and minimum Cd^2.

Design	Generators	max C_d	min C_d^2
2^{6-1}	*F=ABCDE*	1120	56320
2^{7-2}	*F=ABCD, G=BCD*	1664	118784
2^{8-3}	*F=ABDE, G=ACDE, H=ABCD*	2304	235520
2^{9-4}	*F=ABCDE,G=AC,H=AB*	3200	471040
2^{10-5}	*F=BCDE, G=ADE, H=BCE, J=ACDE*	4128	820224
2^{11-6}	*F=ABC, G=ACDE, H=BCDE, J=CDE, K=ABCD*	5120	1325056
2^{12-7}	*F=BDE,G=CD, H= BC, J=ABDE, K=ABC, L=ADE*	6336	2119680
2^{13-8}	*F=BDE, G=ABCD, H= BCD, J=ACD, K=BC, L=ABDE, M=AE*	7552	3178496
2^{14-9}	*F=ABCD, G=ACD, H=BCE, J=CDE, K=AC, L=BCDE, M=ABCDE, N=ACE*	8800	4566016
2^{15-10}	*F=BCDE, G=ABDE, H=ACE, J=BCE, K=ACD, L=AC, M=BC, N= CDE, O= CE*	10112	6397952
2^{16-11}	*F=BCDE, G=ABD, H=ACE, J=BCE, K=ACD, L=AC, M=BC, N=CDE, O=CE, P=ABCDE*	11328	8579072
2^{17-12}	*F=BCDE, G=ABD, H=ACE, J=BCE, K=ACD, L=BDE,M=BC, N=CDE, O=CE, P=ADE, Q=BCD, R=AC*	12416	1105510

Table 6. Best designs in 64 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{7-1}	*G=BCDEF, H=ABCDE,*	3584	229376
	G=ACDEF, H=ABDE		
2^{8-2}	*G=ACDEF, H=ABCE*	5376	589824
	G=BCDEF, H=ABCD		
2^{9-3}	*G=CDEF, H=ABCDE, J=ABCD*	7424	1114112
	G= ABEF, H=ACDE, J=ABCD		
2^{10-4}	*G=ABCDF, H=ACDE, J=BCEF, K=BCDE*	9297	1937408
	G= ABCDF, H=ACDE, J=ABEF, K=ABCDE		
2^{11-5}	*G=ABDEF, H=ACEF, J=CDE, K=ACDF, L=BCDEF*	12800	3268608
	G= ABCDE, H=ACEF, J=ABCE, K=ADEF, L=BCDE		
2^{13-7}	*G=AEF, H=CDE, J=ABCD, K=BCE, L=CDEF, M= ABCF*	16256	5300224
2^{13-7}	*G=ABCDF, H=ACEF, J=BCEF, K=ACDE, L=ACD, M=BCE, N=CDEF*	20224	8470528
2^{14-8}	*G=ABCEF, H=BCDEF, J=ACEF, K=AEF, L=ABCDE, M=BCDE, N=ADEF, O=CDEF*	24512	12472320
2^{15-9}	*G=ABCD,H=ABDE, J=ACEF, K=BCE, L=ABCDE, M=ACDF, N=BCEF, O=CDEF, P=CDE*	29440	18333696
2^{16-10}	*G=ABCDEF, H=ACDE, J=ABCD, K=ABC, L=BDEF, M= BCDE, N=BCD, O=CDF, P=CDE, Q=DEF*	34368	25473024
2^{17-11}	*G=BCDEF, H=ABCDE, J=ABDE, K=ABC, L=BCDF, M=BCDE, N=BCD, O=ACDF, P=ADEF, Q=CDEF, R=CDF*	40704	36306944

Table 7. Best designs in 128 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{8-1}	*H=CDEFG*	10752	1376256
2^{9-2}	*H=ACDEFG, J=ABCDF*	15360	2621440
2^{10-3}	*H=ABEFG, J=ABCF, K=ABCDE*	21120	4669440
	H=ABCFG, J=ABEF, K=ABCDE		
	H=ABCG, J=ABEF, K=ABCDE		
2^{11-4}	*H=CDEFG, J=ADEF, K=ABCDE, L=ABCD*	27648	7602176
2^{12-5}	*H=AEFG, J=ABEF, K=ABCDE, L=ABCD, M= ABCFG*	35584	12222464
2^{13-6}	*H=CDEFG, J=ABDEFG, K=ABDF, L=ABDE, M=BCDG, N=DEFG*	44032	18776064
2^{14-7}	*H=ABDEFG, J=ACEF, K=ABCDE, L=ABCDF, M=CDEFG, N=BCDEF, O=DEFG*	55168	28852224
2^{15-8}	*H=ABEFG, J=ABCF, K=ABCDE, L=ABC, M=BCDEFG, N=BCDEF, O=BCDE, P=BCDFG*	65536	42172416
2^{16-9}	*H=ACDEFH, J=ABFG, K=ABCDEF, L=ACDE, M=ABCD, N=CEFGH, O=DFGH, P=EFGH, Q=BCDG*	76800	60620800
2^{17-10}	*H=ABCFG, J=ABCD, K=ABCDE, L=ABCG, M=BCDEFG, N=BCDEF, O=BCDEG, P=BCDG, Q=CDEF, R=CEF*	94208	84279296

Table 8. Best designs in 256 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{9-1}	J=ABCDEFGH , J=ABCDEFG, J=ABCDEH	30720	7864320
2^{10-2}	J=ABCFGH, K=ABCDE J=ABCDH, K=ABEFGH J=CDFGH, K=ABEFGH	21120	4669440
2^{11-3}	J=ABCDGH,K=ABFG,L=ABCDF	56320	20971520
2^{12-4}	J=BCDEGH, K=ACDFG, L=ABCEF, M=ACDE	73216	29229056
2^{13-5}	J=ABFGH, K=ABCDG, L=CDEF, M=ABDE, N=ABCD J=AEFGH, K=CDEFG, L=ABCDEF, M=ADE, N=ABCD	91136	49414144
2^{14-6}	J=ABCFGH,K=BCDEFG,L=ABCDEF,=ABCDEH,N=ABCDG, O=CDEGH	113408	69140480
2^{15-7}	J=AFGH,K=AFG,L=ABCF,M=ABCDE,N=ABCD,O=CDEH,P=DEFGH	139264	98816529
2^{16-8}	J=ABCH,K=ABFG,L=ABCEF,M=CDE,N=ABCD,O=CFGH, P=DEGH,Q=DEFG	166912	149282476
2^{17-9}	J=ACDEFH, K=ABFG ,L=ABCDEF, M=ACDE ,N=ABCD, O=CEFGH, P=DFGH, Q=EFGH, R=BCDG	199680	188874752

Table 9. Best designs in 512 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{10-1}	K=ABCFGH	84480	43253760
2^{11-2}	K=ABFGHJ,L=ABCDEF	112640	57671680
2^{12-3}	K=ADEFGHJ,L=DEFGH,M=ABCDEF K=AEFGHJ,L=ABCGH,M=ABCDEFG	146432	85458944
2^{13-4}	K=AEFGHJ, L=DEFGH, M=ABCDE, N=ABCDEFHJ	186368	126877696
2^{14-5}	K=ABEFGHJ, L=AFGH M=ABDG, N=ABEF, O=ABCDE	232960	182190080
2^{15-6}	K=ABCDFGHJ, L=BCDEFGH, M=ADEFG,N=ABCDEF, O=ABCDE, P=ABCD	280576	265289728
2^{16-7}	K= ABCEFHJ,L=ABFGH, M=ACEFG ,N= BCEF O=ABCDE, P=ABDHJ, Q=DEGHJ	348160	363331584
2^{17-8}	K= ABCDHJ, L=AEFGH, M=ABCDEFG, N=ADEF ,O=ABCDE, P=BEFGH, Q=BDEFG, R=BCDEGH	409600	505413632

Table 10. Best designs in 1024 runs on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{11-1}	L=ABCDEFHJK L=ABDFGHJK	225280	230686720
2^{12-2}	L=BCDEHJK, M=ABDEFGHJ L=ABCDEK, M=ABCDEFGHJ	292864	299892736
2^{13-3}	L=ABFGHJK, M=ABCDEHJ, N=ABCDEFGH L= ABCDGHJK, M=ABEFGHJ, N=CDEFGH	372736	402653184
2^{14-4}	L=ABCHJK, M=ABCDGHJ, N=ABEFGH, O= ABCDL=ABCDEFK, M=DEFGHJ, N=ABCFGH O=BCDEFG	465920	560988160
2^{15-5}	L=ABCDGHJK, M=CDEFGHJ, N=ABEFGH, O=ABCFG, P=CDEF	573440	734003200
2^{16-6}	L=ACDEFGHJK, M=ABDEFGHJ, N=ABCFH, O=ABCG, P=ABCDE, Q=AGHJK L=AEFGHJK, M=ABDFGHJ, N=ABCGHJK, O=ABCEG, P=ABDEFG, Q=AGJK	696320	692060160
2^{17-7}	L=AFGJK, M=ABEGHJ, N=ACDEFGH, O=ABCFG, P=ABCDEF, Q=ABCDE, R=ABCD L=AFGHJK, M=ABEFGHJ, N=ABCDEFGH, O=ABCFG, P=ABCDEFK, Q=ABCDE, R=ABCDGHJ L=ACDFGHJK, M=ABEFJ, N=ABFGH, O=ABCFG, P=ABCDEF, Q=ABCDEJK, R=ABCDGHJ	827392	987758592

Table 11. Simulated best designs in two-factor interactions on the basis of maximum C_d and minimum C_d^2 .

Design	Generators	max C_d	min C_d^2
2^{5-1}	*E=ABCD*	320	10240
2^{6-1}	*F=ABCDE*	1120	56320
2^{7-1}	*G=ABCDEF*	3584	229376
2^{7-2}	*F=ABCD, G=BCD*	1664	118784
2^{8-2}	*G=CDEF, H=ABCDE*	5376	589824
2^{9-2}	*H=ACDEFG, J=ABCDF*	15360	2621440
2^{9-3}	*G=CDEF,H=ABCDE,J=ABCD*	7424	1114112
2^{8-1}	*H=CDEFG*	10752	1376256
2^{10-1}	*K=ABCFGH*	84480	43253760
2^{11-3}	*J=ABCDGH, K=ABFG, L=ABCDF*	56320	20971520
2^{11-2}	*K=ABFGHJ, L=ABCDEF*	112640	57671680
2^{12-3}	*K=ADEFGHJ, L=DEFGH, M=ABCDEF*	146432	85458944
2^{12-2}	*L=BCDEHJK, M=ABDEFGHJ* *L=ABCDEK, M=ABCDEFGHJ*	292864	299892736
2^{13-4}	*K=AEFGHJ, L=DEFGH, M=ABCDE, N=ABCDEFHJ,*	186368	126877696
2^{14-5}	*K=ABEFGHJ, L=AFGH M=ABDG, N=ABEF, O=ABCDE*	232960	182190080
2^{14-4}	*K=ABCHJK, L=ABCDGHJ, M=ABEFGH N= ABCD* *K=ABCDEFK, L=DEFGHJ, M=ABCFGH N=BCDEFG*	465920	560988160
2^{15-5}	*L=ABCDGHJK, M=CDEFGHJ, N=ABEFGH, O=ABCFG, Q=CDEF*	573440	692060160
2^{16-6}	*L=ACDEFGHJK, M=ABDEFGHJ, N=ABCFH, O=ABCG, P=ABCDE, Q=AGHJK* *L=AEFGHJK, M=ABDFGHJ, N=ABCGHJK, O=ABCEG, P=ABDEFG, Q=AGJK*	696320	987758592
2^{17-7}	*L=AFGJK, M=ABEGHJ, N=ACDEFGH, O=ABCFG, P=ABCDEF, Q=ABCDE, R=ABCD* *L=AFGHJK, M=ABEFGHJ, N=ABCDEFGH, O=ABCFG, P=ABCDEFK, Q=ABCDE,R=ABCDGHJ* *L=ACDFGHJK, M=ABEFJ, N=ABFGH, O=ABCFG, P=ABCDEF, Q=ABCDEJK, R=ABCDGHJ*	827392	1505755136

RESULTS AND DISCUSSION

The (M, S)-optimality criterion is used for regular FFD's. The proposed formulae for three-factor interactions were used for simulations study and found the best designs on the basis of trace C_d and trace C_d^2 in different runs from 8, 16, 32, 64, 128, 256, 512, and 1024 up to 17 factors by using different generators. Tables 3,4,..,10 show that these designs are selected as the best designs on the basis of maximum trace C_d and minimum trace C_d^2. The generators, which are used, have best alias structures. The Table 11 shows the designs which have defining relations which can be applied on all main effects, all two-factor interactions and some three-factor interactions. These designs consist of 2^{5-1},2^{6-1}, 2^{7-1} , 2^{7-2} ,2^{8-2}, 2^{9-2}, 2^{9-3} ,2^{8-1} , 2^{10-1} , 2^{11-3} , 2^{11-2} , 2^{12-3} , 2^{12-2} , 2^{13-4} , 2^{14-5} , 2^{14-4}, 2^{15-5}, 2^{16-6}, 2^{17-7} and different generators have been used, but the selected generators have maximum trace C_d and minimum trace C_d^2 within the chosen class. The table 12 reveals that the designs consist of 2^{5-1}, 2^{6-1}, 2^{7-1}, 2^{7-2}, 2^{8-2}, 2^{9-2}, 2^{9-3}, 2^{10-3}, 2^{10-2} , 2^{11-4}, 2^{12-4}, 2^{13-6}, 2^{13-5}, 2^{14-7}, 2^{14-6}, and their generators are applied for all main effects, all two-factor interactions and all three-factor interactions. Tables 11 and

12 have the maximum trace C_d and minimum trace C_d^2 and all effects can be tested for the trace C_d and trace C_d^2.

(M, S)-optimality criterion is also important to apply on many designs where higher order interactions are also of interest. The (M, S)-optimality criterion is simpler in computation, but it is also self-sufficient of the selection of orthonormal contrasts while the main effects, 2*fi*'s and 3*fi*'s, are focused in this study. Regular FFD's and two components of the (M, S) criterion, i.e., trace C_d and trace C_d^2, are derived as exclusively functions of the numbers of three-letter and four-letter words. Usually, the designs under this criterion are not MA designs, but all MA designs up to 64 runs are (M, S)-optimal (Qu *et al.,* 2008).

Table 12. Simulated best designs in three-factor interactions on the basis of maximum C_d and minimum C_d^2.

Design	Generators	max C_d	min C_d^2
2^{5-1}	*E=ABCD*	320	10240
2^{6-1}	*F=ABCDE*	1120	56320
2^{7-1}	*G=ABCDEF*	3584	229376
2^{7-2}	*F=ABCD,G=BCD*	1664	118784
2^{8-2}	*G=CDEF, H=ABCDE*	5376	589824
2^{9-2}	*H=ACDEFG, J=ABCDF*	15360	2621440
2^{9-3}	*G=CDEF, H=ABCDE, J=ABCD*	7424	1114112
2^{10-3}	*H=ABEFG, J=ABCF, K=ABCDE* *H=ABCFG, J=ABEF, K=ABCDE* *H=ABCG, J=ABEF, K=ABCDE*	21120	4669440
2^{10-2}	*J=ABCFGH, K=ABCDE* *J=ABCDH, K=ABEFGH* *J=CDFGH, K=ABEFGH*	42240	12124160
2^{11-4}	*H=CDEFG, J=ADEF, K=ABCDE, L=ABCD*	27648	7602176
2^{12-4}	*J=BCDEGH, K=ACDFG, L=ABCEF, M=ACDE*	73216	29229056
2^{13-6}	*H=CDEFG, J=ABDEFG, K=ABDF, L=ABDE,M=BCDG, N=DEFG*	44032	18776064
2^{13-5}	*J=ABFGH, K=ABCDG, L=CDEF, M=ABDE, N=ABCD*	91136	49414144
2^{14-7}	*J=AEFGH, K=CDEFG, L=ABCDEF, M=ADE, N=ABCD*	55168	28852224
2^{14-6}	*H=ABDEFG, J=ACEF, K=ABCDE, L=ABCDF, M=CDEFG, N=BCDEF, O=DEFG* *J=ABCFGH, K=BCDEFG, L=ABCDEF, M=ABCDEH,N=ABCDG, O=CDEGH*	113408	69140480

CONCLUSION

Tables 3, 4…10 show some selected designs among the many designs simulated by R- package with different generators. The simulated designs consist of generators which have the best alias structures. The selected designs are the best designs on the basis maximum trace C_d and minimum trace C_d^2 which is obtained for using generators and their defining relations. It is shown from the table 10 in design 7, 2^{17-7} seven generators are *L=AFGJK, M=ABEGHJ, N=ACDEFGH, O=ABCFG, P=ABCDEF, Q=ABCDE, R=ABCD and L=ACDFGHJK, M=ABEFJ, N=ABFGH, O=ABCFG, P=ABCDEF, Q=ABCDEJK, R=ABCDGHJ* and have same trace C_d which is 831488 but different trace C_d^2 1505755136 and 1317011456 respectively. According the (M, S)-optimality criterion if both design have same trace C_d then we will select the design having minimum trace C_d^2 from that class of designs. So in this scenario the design 2 is better than design 1 on the basis of minimum trace C_d^2 which is 1317011456. Mostly, lower order generators have less trace C_d and maximum trace C_d^2. Tables 11 and 12 show that the designs have higher runs, but one can use them for large size experiments if it can afford it. We can use up to 17 factors designs and up to 1/11 fractions. It is also found that some designs have two or three different type of generators which have same trace C_d, but different trace C_d^2.

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